

Granular Casimir effect and depletion induced phenomena in mixtures of rods and spheres

Luis A. García-Trujillo

*Facultad de Ciencias Físico-Matemáticas, Universidad Autónoma de Coahuila,
Edificio D, Unidad Camporredondo, Saltillo, Coahuila, Mexico*

Gustavo M. Rodríguez-Liñán,* Jorge F. Reyes-Tendilla, and Yuri Nahmad-Molinari
*Instituto de Física “Manuel Sandoval Vallarta”, Universidad Autónoma de San Luis Potosí,
Álvaro Obregón No. 78, 78000, San Luis Potosí, San Luis Potosí, Mexico*

Gabriel Pérez-Ángel

*Departamento de Física Aplicada, Centro de Investigación y de Estudios Avanzados del IPN,
Unidad Mérida, AP 73 “Cordemex”, 97310, Mérida, Yucatán, Mexico*

(Dated: December 14, 2012)

A novel phenomenon analogous to the quantum and critical Casimir effects is observed in a quasi-2D vertically shaken granular gas of rods and spheres. This granular Casimir attraction increases as the shaking intensity or the rod length is increased. Velocity distributions of spheres inside and outside the Casimir-like configurations are measured, showing a clear suppression of the momentum acquired by the spheres in direction perpendicular to the rods due to inelastic collapse. Excluded volume effects are measured through liberated volume as a function of time. The apparition of stable Casimir-like configurations instead of compact hexagonal aggregates reveals a stronger attraction than the one purely induced by depletion forces. Furthermore, evidence of layering of spheres close to the boundaries (resembling the ordering of water on a surface) is reported.

PACS numbers: 45.70.-n, 45.70.Qj, 05.40.-a

Since 1948 when Casimir predicted the attraction between two flat conducting surfaces due to quantum fluctuations of the zero field of vacuum, a thorough experimental search was initiated [1]. It was not until four decades after, when Lamoreaux in 1997 [2], first conclusively demonstrated the Casimir effect. Subsequently, it was theorized by Fisher and de Gennes [3] that substituting electromagnetic quantum fluctuations by density fluctuations of the medium (for instance, in a binary mixture close to the critical point) would conduce to an analogous phenomenon called from then on the critical Casimir effect. This was directly measured in 2008 by Hertlein by means of frustrated total internal reflection microscopy techniques [4]. More recently, Reza Shaeabani [5], following the predictions made by Brito and Cattuto of a granular Casimir effect [6, 7], found numerically a non additive character of such granular Casimir forces not yet observed experimentally.

On the other hand, Asakura and Oosawa, in the fifties [8], investigated the role of small colloids in the aggregation of larger ones and developed a theory to describe the attraction between large particles due to depletion of small particles within the gap in between the larger ones. For this reason, these forces are sometimes called depletion forces, but are referred as well as excluded volume forces. They are behind the entropically driven ordering produced in colloids by adding some depleting agent (usually a polymer or a macromolecule). Depletion forces have been broadly investigated at colloidal [9] and even at granular scales [10–14]. Furthermore, this depletion forces can produce a layering effect at molecular or colloidal scales [15], not yet observed in granular materials [16].

Both kinds of forces (Casimir and depletion) share in common that there exists a region in between large objects, im-

mersed in a sea of small fluctuating entities, that becomes depleted from momentum transfer coming from such tiny entities (being electromagnetic, mechanical or of some other nature). This imbalance of pressure from the interior and the exterior region of the large structures give rise to the effective attraction.

In this paper, we report both kinds of attraction (excluded volume and Casimir forces) between granular rod-like particles immersed in a 2-D gas of spheres. Subtle differences make possible to discern which effect is behind each phenomenon, since both are coupled and enhance each other.

Different granular mixtures of brass rods (2.5 cm in length and 3.3 mm in diameter) and steel or lead spheres of the same diameter of the rods are vertically shaken using a sinusoidal signal of frequency 60 Hz and at an amplitude of 0.24 mm, on top of a flat horizontal plate. Digital video (Red Lake Motion Meter camera, 500 fps) is recorded from above and the position of each particle is determined as it evolves. Starting from a configuration in which rods are homogeneously distributed on the plate (Fig. 1 (a)), the evolution of these rods immersed in the ocean of spheres is recorded and analyzed *a posteriori* by means of an *ImageJ* software routine.

The mixture evolves from a homogeneous distribution of rods, set initially by hand, towards islands of aggregated rods that stick together laterally, forming dimmers, trimmers or polymers, which eventually tend to migrate close to a wall of the cell. The force that allows dimmer formation is the depletion force originated by the fact that if two rods meet each other laterally, the region in between them is depleted of spheres and therefore, there is no momentum transfer to the rods from this “interior” region, unbalancing the momentum transfer produced from the spheres outside. In Fig. 1 (a) and

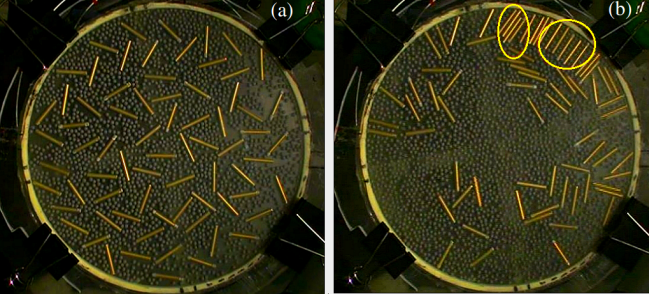


FIG. 1. (a) Initial and (b) final configurations of a mixture of rods and spheres after evolving for five minutes.

(b), a typical initial and final (after five minutes) configurations are shown respectively.

As can be seen from the snapshot of Fig. 1 (b), a phase separation occurs and several aggregates form after the experiment starts. The two new phases are constituted by a gas of spheres with just few rods diluted in it and several islands of aggregates of parallel rods. These aggregates can be of two, three or more parallel rods in close contact or two parallel rods sandwiching a chain of spheres in between. Two representative examples of such configurations are highlighted on the picture. The rods stuck together in close contact will be referred as depletion aggregates while the configuration of sandwiched spheres will be called a Casimir aggregate. The main difference among both kinds of aggregates resides in the fact that the region in between two successive parallel rods is depleted or not from roaming spheres. At a first glance, one can attribute the formation of Casimir-like configurations to depletion forces as well. However, a closer analysis reveals that the density of spheres per unit area is always higher in the interior region (at least by a ten percent) than outside the rods, leading, in principle, to a higher “osmotic” pressure from inside the configuration that would produce an effective force pushing apart the rods. This, in turn, would be observed as an effective repulsion of rods whenever a set of spheres are trapped in between them, making the observed configuration unstable. On the contrary, these structures are very likely to be produced and are stable enough to subsist in stationary state during the whole experiment once formed.

The mechanism behind the stability of Casimir aggregates can be unveiled by means of studying the velocity distributions of spherical particles inside and outside the structure. This was done by zooming-in close to a single Casimir configuration and measuring the displacements of the spherical particles among consecutive frames. In Fig. 2, we show comparatively the resulting velocity distributions for particles sandwiched between rods and for particles in the surrounding gas of spheres. The suppression of momentum acquisition proceeds in both directions — perpendicular and parallel to the rods — since the sandwiched particles lose momentum due to a phenomenon similar to the inelastic collapse effect when they collide against one another [17].

A clear suppression of the perpendicular component of the

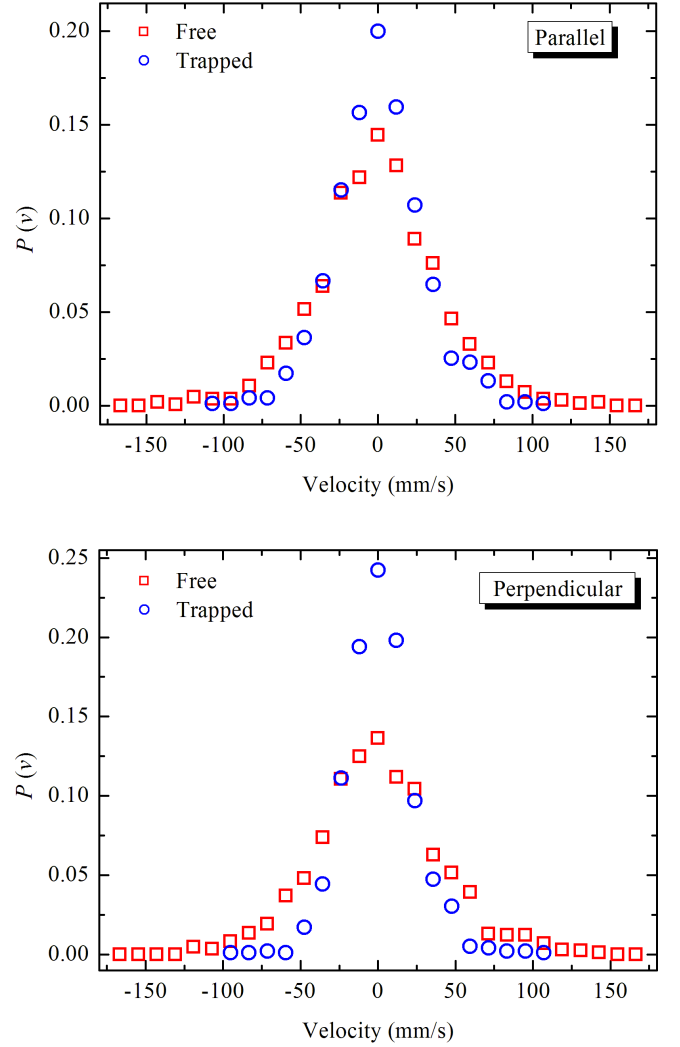


FIG. 2. Probability distributions for velocity components along parallel and perpendicular directions to the confining rods for Casimir configurations. These are shown in blue circles for sandwiched spheres and in red squares for untrapped particles belonging to the fluid phase.

movement is found for those particles trapped between rods in the Casimir configuration, showing that their corresponding kinematic modes are inhibited in exactly the same way as it occurs with electromagnetic modes within the interior region defined by the conducting plates in the quantum-mechanical Casimir effect. In the quantum Casimir effect, the quantization of modes required by the boundary conditions imposed by the conducting plates, shaves the electromagnetic field spectrum within the interior region but does not affect the zero-point fluctuating field spectrum outside the plates, producing an imbalance of radiation pressure that exerts a net attractive force between the conducting plates. Analogously, the particles sandwiched between rods, are less likely to acquire momentum in the direction perpendicular to the confining rods, and are only prompted to acquiring momentum from the plate in the vertical direction and along the rods. This is the reason

why we have called these configurations Casimir aggregates.

This mechanical inhibition of momentum acquisition perpendicular to the rods is due to the fact that spherical particles that eventually find themselves in between two rods will suffer a dramatic rate of energy loss due to a very large number of collisions per unit time with the caging rods, in close resemblance to the inelastic collapse that Goldhirsch and Zanetti have reported since 1993 [17]. This quasi inelastic collapse is the main responsible of inhibiting the acquisition of momentum perpendicular to the rods and consequently the origin of the granular Casimir effect reported here.

As we have stated above, the larger number density of trapped particles within the couple of confining rods with respect to the surrounding gas, would induce an effective repulsive force between the rods making the Casimir configuration unlikely. Furthermore, one can expect that the stability of such configuration would decrease as the amplitude of the velocity distribution for the surrounding gas spheres is increased, since the rods as well are subjected to these velocity fluctuations and consequently they would have increased translational and rotational diffusion. However, the granular Casimir force will behave the other way around, as predicted theoretically by Brito and Cattuto [6, 7]. This means that the force between rods, and thus the stability of the aggregates, should increase as the shaking strength Γ_{fluc} does.

The relationship among the Casimir force per area unit and the fluctuation intensity (in our case the shaking acceleration) for a system of two large immobile spheres in a sea of smaller moving spheres, can be expressed in the form [6]:

$$\frac{F}{A} = \Gamma_{\text{fluc}} k_0 \frac{\partial^2 p}{\partial n^2} \frac{1}{16\pi D} \left[1 - \frac{\log(\sinh l)}{l} \right], \quad (1)$$

where $l = k_0 L_x$ is a reduced distance with respect to the separation between plates L_x and k_0 a characteristic length of the system. D is the diffusion coefficient and p and n the pressure exerted by the particles and the number density of particles respectively. Thus, the Casimir force will growth linearly with Γ_{fluc} .

This last prediction is corroborated indirectly by increasing the shaking amplitude and measuring the stability (that is, the survival times) of a single Casimir configuration set as the initial condition. Twenty measurements were made for each shaking amplitude. The resulting survival times are shown in Fig. 3 as a function of the dimensionless acceleration Γ . It is worth to note that depletion dimmers could last up to tens of minutes, which give us an idea of how much more stable they are in comparison with Casimir-like structures.

Entropically driven ordering of rods is due to the fact that rods in contact have smaller excluded area than separated rods. In other words, as rods stick in close contact (reducing the entropy of the system) the spheres have a larger area to roam and consequently increase their number of possible configurations and thus, the whole entropy of the system. Here, the liberated area is the difference between the excluded areas of two independent rods and the excluded area of two rods in

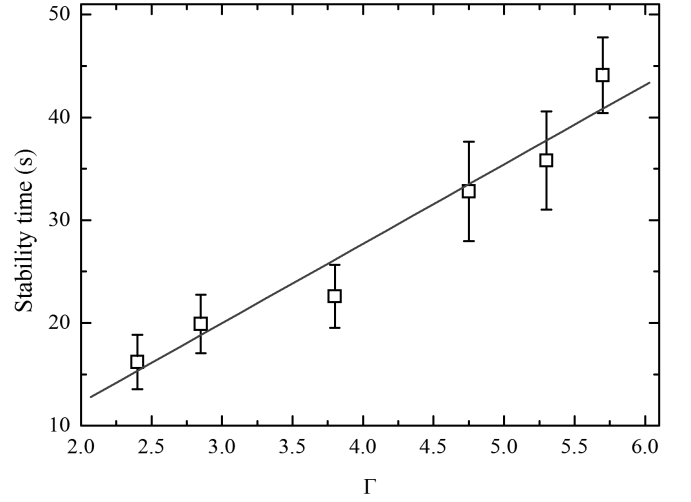


FIG. 3. Stability (survival times) of Casimir-like configuration as a function of dimensionless acceleration (fluctuation intensity), showing a monotonically growing dependence.

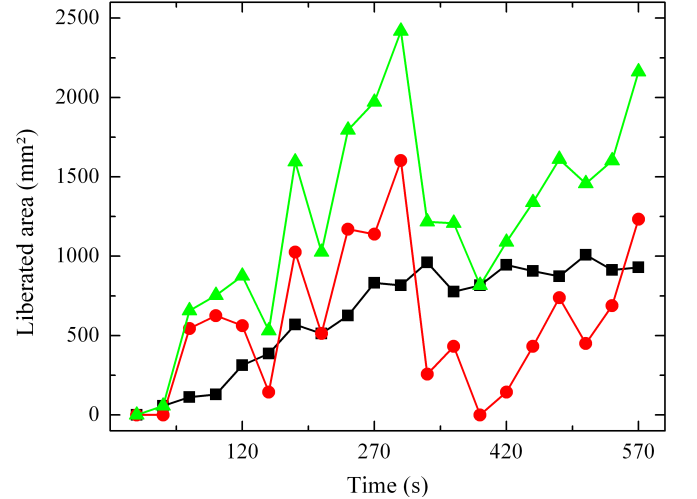


FIG. 4. Liberated area due to aggregation of rods for depletion dimmers (black squares), Casimir like configurations (red circles), and total liberated area by both kinds of aggregation mechanisms (green triangles), as a function of time.

contact. For purposes of counting the effect of Casimir configurations in increasing the area available for roaming, just the overlapped area of spheres within two rods in a Casimir configuration is taken into account. In Fig. 4, liberated area due to depletion (black squares), Casimir (red circles) structures, and total liberated area are plotted as a function of time for an area fraction coverage of 0.74, including 72 rods. A smooth growing behavior of liberated volume corresponding to depletion pairs reflects the phase separation of rods from the gas of spheres. It contrasts with the strong fluctuating behavior of the liberated area of Casimir-like configurations. This fluctuating behavior results from the lower stability (magnitude of the force) compared with depletion forces acting between

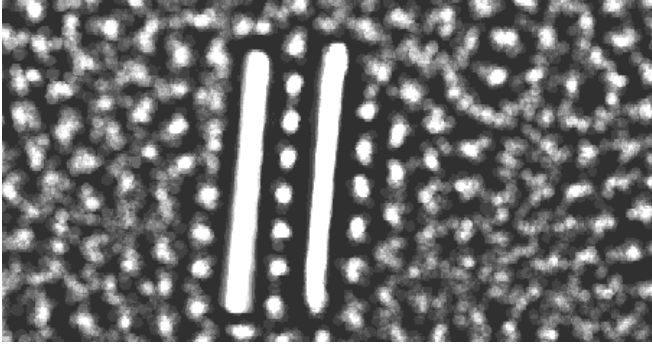


FIG. 5. Superimposed pictures of spheres in the interior region of a Casimir aggregate and its vicinity for rods of 2.5 cm.

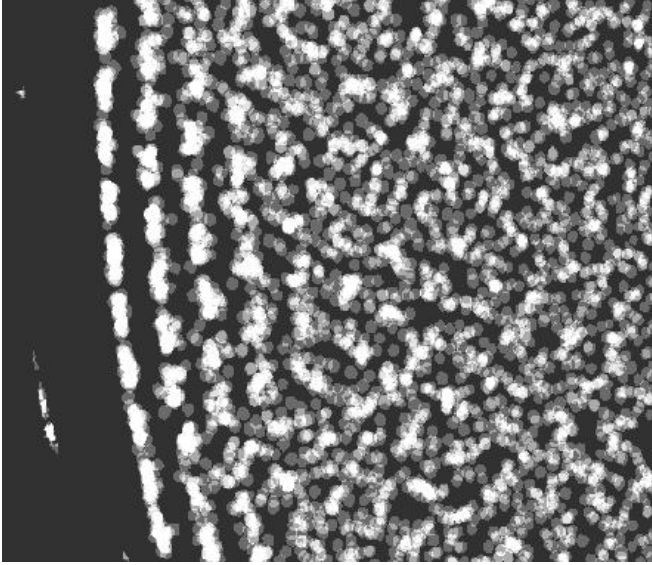


FIG. 6. Superimposed pictures of spheres in the region close to the border of the plate.

rods.

Furthermore, an analogous of the wetting and layering phenomenon that occurs for molecular and colloidal systems [15], mainly driven by depletion forces and inelastic collapse appears in our experiments. In Fig. 5 and 6, two sets of 20 superimposed series of pictures taken at intervals of 20 ms show this granular wetting. In Fig. 5, a typical Casimir pair is depicted in which the suppression of movement perpendicular to the rods for the trapped particles can clearly be distinguished, and a gradient of mobilities from the external surface of the rod towards the bulk of the gassy phase. In Fig. 6, the superimposed pictures were taken close to the border of the plate, strikingly showing the layering and the increasing mobility of

particle towards the plate center.

In conclusion, we have investigated a granular 2D gas composed of a mixture of inelastic rods and spheres in which segregation proceeds via the increase in entropy by excluded area reduction. Area is liberated when two or more rods join side to side or when Casimir-like configurations are created. Besides, these Casimir structures suppress, for the spheres trapped within, their ability of acquiring momentum in the direction perpendicular to the rods, by a mechanism analogous to inelastic collapse, provoking a pressure imbalance between the interior and the exterior region of the configuration leading to an effective net attraction among rods. This is shown through velocity distributions in direction parallel and perpendicular to the rods for spheres with and without the Casimir configuration. Finally, survival times grow linearly with the shaking amplitude supporting the identification of these structures as caused by a granular Casimir effect.

Thanks are given for the founding of this research to the grant CONACyT 82975 and a scholarship for G. M. Rodríguez-Liñán.

* gsrzdl@ifisica.uaslp.mx

- [1] H. G. B. Casimir, Proc. Kon. Ned. Acad. **51**, 793 (1948)
- [2] S. K. Lamoreaux, Phys. Rev. Lett. **78**, 5 (1997)
- [3] M. E. Fisher and P. G. de Gennes, C. R. Acad. Sci. Paris B **287**, 207 (1978)
- [4] C. Hertlein, L. Helden, A. Gambassi, S. Dietrich, and C. Bechinger, Nature (London) **451**, 172 (2008)
- [5] M. Reza Shaeabani, J. Sarabadani, and D. E. Wolf, Phys. Rev. Lett. **108**, 198001 (2012)
- [6] R. Brito, R. Soto, and U. Marconi, Granular Matter **10**, 29 (2007), ISSN 1434-5021
- [7] C. Cattuto, R. Brito, U. M. B. Marconi, F. Nori, and R. Soto, Phys. Rev. Lett. **96**, 178001 (2006)
- [8] F. Oosawa and S. Asakura, J. Chem. Phys. **22**, 1255 (1954)
- [9] H. N. W. Lekkerkerker and R. Tuinier, eds., *Lecture Notes in Physics, Berlin Springer Verlag*, Lecture Notes in Physics, Berlin Springer Verlag, Vol. 833 (2011), and references therein
- [10] M. Bose, U. U. Kumar, P. R. Nott, and V. Kumaran, Phys. Rev. E **72**, 021305 (2005)
- [11] P. Melby, A. Prevost, D. A. Egolf, and J. S. Urbach, Phys. Rev. E **76**, 051307 (2007)
- [12] D. A. Sanders, M. R. Swift, R. M. Bowley, and P. J. King, Phys. Rev. Lett. **93**, 208002 (2004)
- [13] J. Duran and R. Jullien, Phys. Rev. Lett. **80**, 3547 (1998)
- [14] J. Galanis, R. Nossal, and D. Harries, Soft Matter **6**, 1026 (2010)
- [15] J. N. Israelachvili and R. M. Pashley, Nature (London) **306**, 249 (1983)
- [16] T. Kruppa, T. Neuhaus, R. Messina, and H. Löwen, J. Chem. Phys. **136**, 134106 (2012)
- [17] I. Goldhirsch and G. Zanetti, Phys. Rev. Lett. **70**, 1619 (1993)